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A 3D SHAPE RETRIEVAL METHOD BASED ON CONTINUOUS SPHERICAL WAVELET TRANSFORM

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ABSTRACT
Recently, many efforts have concentrated on finding efficient content based retrieval methods of 3D objects. In this paper, we proposed a new retrieval method. The method is constructed on a shape descriptor based on continuous spherical wavelet transform.

Continuous 2D wavelet transform has extinct advantages in content based image retrieval. The continuous wavelet transform can be extended from two dimensions to more dimensions, for example, spherical space, with the same properties. As a natural extension, continuous spherical wavelet transform can realize a spherical analysis. Therefore, we map a shape into a unit sphere by spherical parameterization, followed by continuous spherical wavelet transform of the spherical function. This method is our contribution. The result of the transform can be as a new descriptor and be used to match the dissimilarity of two shapes. We have examined our method on a small database of general objects and it is confirmed to be efficient.

KEY WORDS
3D object, content based retrieval, shape descriptor, continuous spherical wavelet transform

1. Introduction

With the recent rapidly increasing of 3D data in many applications, such as computer games, computer aided design, VR environments, biology, e-business, etc., 3D shape data retrieval and reutilization of them becomes more important.

Accordingly, there is an increasing need for computer algorithms to help people find their interesting 3D shape data and discover relationships between them. Recently, many efforts have concentrated on researching techniques for efficient content based retrieval of 3D objects [1].

The key of a content based retrieval is to develop a descriptor capturing the main feature of 3D objects, because 3D shapes can be discriminated by measuring and comparing their features. A descriptor is a *d*-dimensional vector of values, and as for all the 3D shapes, the dimension *d* is fixed. In the *d*-dimensional space, if two vectors are close, two shapes are considered to be similar.

Because 3D model retrieval has some similar characters with image retrieval, many 3D model retrieval methods become an extension from image retrieval methods. Wavelet theory has greatly developed in computer graphics. Continuous 2D wavelet transform plays an important role in image analysis and retrieval [21, 23]. The continuous wavelet transform can be extended from two dimensions to more dimensions, for example, spherical space, with the same properties. As a natural extension, continuous spherical wavelet transform can realize a spherical analysis.

In this paper we present a 3D content based retrieval method relying on a shape descriptor based on continuous spherical wavelet transform. The shape descriptor is computed as the following:

1. Pose estimation: Translate the centroid of all the models into the origin of the coordinate system and use the Principal Component Analysis (PCA) method to get the rotation invariant dissimilarity measures.
2. Spherical parameterization: A cluster of rays are cast from the centroid of the model, and intersect with surfaces of the model. A spherical function can be defined by the distances of the farthest intersections from the centroid of the model as a function of latitude and longitude.
3. Continuous Spherical Wavelet Transform: Perform the Continuous Spherical Wavelet Transform (CSWT) of the spherical function according to CSWT theory introduced by J-P Antoine[2]. To our knowledge, this theory has not been applied to the content based retrieval of 3D objects so far.
4. Descriptor: The result of the continuous spherical wavelet transform is used as a shape descriptor. The features of models are compared with the shape descriptors.

We apply our method to a small database of general objects collected from free Princeton Shape Benchmark Database [3]. This method is confirmed to be efficient.

The outline of the rest of the paper is as follows: in the next section we shortly review the previous work in 3D model retrieval and relevant spherical wavelet theory. In section 3 we present a general theoretical framework for the new shape descriptor based on continuous spherical wavelet transform. We put emphasis on the analysis of the continuous spherical wavelet transform. Section 4 shows the new shape descriptor and gives a computation method of the descriptor. In Section 5 we present our experimental results and conclude in Section 6.

2. Previous Work

In this section, we will discuss the latest 3D retrieval methods and divide them into two main categories of shape matching, feature distribution and shape descriptors. Finally, we will introduce the relevant development of spherical wavelet theory.

2.1 Feature distribution

It is easy and fast to compare feature distributions of models, and it does not need any normalization of a 3D mesh-model.

Osada et al. [5] match 3D models with shape distribution. The key idea of this method is to present the signature of an object as a shape distribution sampled from a shape function measuring global geometric properties of an object. X.Liu et al. [6] utilize the directional histogram model to characterize 3D shapes. The shape can be described by the thickness distribution in the directions per latitude and longitude. Ohbuchi et al. [7] construct the shape histograms using the moment of inertia about the principal axes of the model.

All the feature distribution methods have a common limitation that these methods can only catch the similar gross shape properties and be powerless to catch the detailed shape properties. The following shape descriptors improved it.

2.2 Shape descriptors

It is considered to be a good solution that the shape is mapped into a sphere with center at the origin, and the shape can be represented by a spherical function, and then, the function can be easily analyzed in a spherical space. In this point, it is similar to our shape descriptor.

As a representative example, spherical harmonics is applied in a large field such as earth physics, image analysis, biology, and so on. The theory of spherical harmonics is analyzed and interpreted by D. Healy in [8]. It is firstly introduced in the 3D model retrieval by D. V. Vranic in [9]. Funkhouser et al. [10] profit from the invariance properties of spherical harmonics and present an affine invariant descriptor based on spherical harmonics. D. Saupe [11] constructed moment-based descriptor by representing a spherical function using spherical harmonics.

The feature extraction is performed using a rendered perspective projection of the object on an enclosing sphere in [12]. It is considered as a shading-based shape descriptor.

Novotni and Klein [13] present a so-called 3D Zernike descriptor by computing 3D Zernike descriptors from voxelized models as natural extensions of spherical harmonics based descriptors.

2.3 Development of spherical wavelet theory

In the last several years, wavelet analysis has been proven useful in many applications. Many efforts have been put into extending wavelets to the unit sphere. Several wavelet transforms on the sphere have been proposed.

Schroder and Sweldens [14] have developed a discrete orthogonal wavelet transform on the sphere. Hamid [22] developed a 3D models retrieval method based on this theory. This discrete orthogonal wavelet transform has a fast speed, however, it is based on the Haar wavelet function which then suffers from the poor properties of the Haar function and the problems inherent to the orthogonal decomposition according to [15].

Recently, a new consistent and satisfactory framework for spherical wavelets have been proposed by J-P Antoine [2]. This framework differs entirely from the discrete wavelet transform. In this framework, a "continuous" wavelet transform on the 2-sphere has been firstly introduced, using group theoretic principles and the general construction of coherent states on manifolds developed in [16].

We firstly introduce this continuous spherical wavelet transform into shape retrieval.

3. Theory Analysis of Continuous Spherical Wavelet Transform

3.1 Definition

To carry out a wavelet analysis of spherical function on a unit sphere, Euclidean wavelet analysis must be extended to spherical wavelet. We refer to the CSWT constructed by J-P. Antoine & L.Demanet [4]. This CSWT is based on a group theoretical approach produced by J-P. Antoine and P.Vander gheynst [2], and constructed on the general construction of coherent states on manifolds developed by S.T. Ali [16].

Constructing a wavelet analysis on the sphere is similar to the construction of Euclidean planar wavelet analysis. Euclidean motions and dilations of mother wavelet are extended to spherical circumstance.

For extending Euclidean motions into a sphere, it is reasonable to consider the rotations as the motions on a sphere. The rotations are defined by the elements of the Rotation Group $SO(3)$, and motions are realized by rotations $\rho \in SO(3)$. A function is defined on a 2-sphere, $f(\omega) \in L^2(S^2, d\Omega)$, $\omega \in S^2$. $L^2(S^2, d\Omega)$ is the square integrable space on the unit sphere, and $d\Omega = \sin\theta d\theta d\varphi$ is the rotation invariant measure on the sphere. Here $\omega \equiv (\theta, \varphi)$, denotes spherical coordinates with latitude θ and longitude φ, and $\theta \in (0, \pi]$, $\varphi \in (0, 2\pi]$. The rotation of the function is defined in the Eq. (1),

$$(R_\rho f)(\omega) = f(\rho^{-1}\omega) \tag{1}$$

The dilations around the North Pole are obtained by considering usual dilations in the tangent plane at the North Pole and lifting them to S^2 by inverse stereographic projection from South Pole, represented in Figure 1.

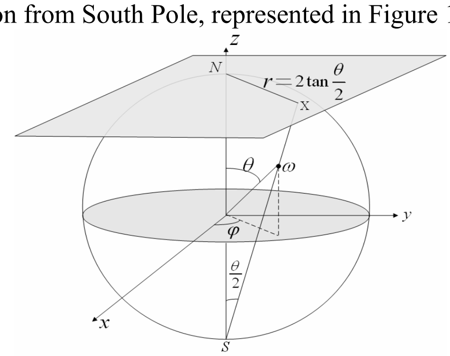


Figure 1. Stereographic projection of the sphere on to the plane

Accordingly, a spherical dilation D_a of a function $f(\omega)$ is defined by Eq.(2).

$$(D_a f)(\omega) = f_a(\omega) = \lambda(a,\theta)^{1/2} f(\omega_{1/a}), \quad a \in R^+ \tag{2}$$

where a is the scale of wavelet, and can control the finer or coarse representation of a signal on a sphere, and $\omega_a \equiv (\theta_a, \varphi)$, $\tan\frac{\theta_a}{2} = a\tan\frac{\theta}{2}$, indeed, $\theta \to \theta_a$ is the dilation obtained by inverse stereographic projection. Here, $\lambda(a,\theta)$ is Radon-Nikodym derivative, and given by

$$\lambda(a,\theta) = \frac{4a^2}{\left[\left(a^2-1\right)\cos\theta + (a^2+1)\right]^2} \tag{3}$$

The wavelet analysis on the sphere is performed by constructing a set of wavelet basis functions by rotations and dilations of an admissible mother spherical wavelet ψ. The formed family $\left\{\psi_{a,\rho} \equiv R_\rho D_a \psi, \rho \in SO(3),\ a \in R^+\right\}$ is an over-complete set of functions in $L^2(S^2, d\Omega)$.

Accordingly, as for an arbitrary spherical function signal $s \in L^2(S^2, d\Omega)$, the CSWT is given by projecting the signal onto the wavelet family, defined as

$$W_\psi^s(a,\rho) = \langle \psi_{a,\rho}, s \rangle = \int_{S^2} (R_\rho D_a \psi)^*(\omega) s(\omega) d\Omega \tag{4}$$

where the $*$ denotes complex conjugation.

3.2 Choice of mother spherical wavelet

A wavelet basis previously described on the unit sphere may be constructed by rotations and dilations of an admissible mother spherical wavelet. The mother spherical wavelet is simply constructed by projecting a 2-D Euclidean mother wavelet on to the sphere by an inverse stereographic projection described in Section 3.1.

2D Euclidean Morlet Wavelet [17] has a good feature in image analysis, and in our paper we decide to adopt the Spherical Morlet Wavelet as the mother spherical wavelet. The Spherical Morlet Wavelet can be constructed by projecting the admissible 2-D Euclidean Morlet Wavelet on to the sphere by an inverse stereographic projection.

2-D Euclidean Morlet Wavelet is defined by

$$\psi_M(\vec{\mathbf{x}}) = \exp(\mathrm{i}\vec{\mathbf{k}}_0 \cdot \vec{\mathbf{x}}) \exp(-\|\vec{\mathbf{x}}\|^2) \tag{5}$$

and the form of Spherical Morlet Wavelet is constructed as the following Eq.(6). Refer to [4] for constructing method.

$$\psi_M(\theta,\varphi) = \frac{\exp(ik_0 \tan\frac{\theta}{2}\cos(\varphi_0 - \varphi))\exp(-\frac{1}{2}\tan^2\frac{\theta}{2})}{1+\cos\theta} \tag{6}$$

The parameter φ_0 is the direction of a wavelet, and in our paper we let $\varphi_0 = 0$. The parameter k_0 is the wave vector of the wavelet, and decides whether the function is admissible, and if k_0 is large enough, typically greater than 6, the function is admissible.

In this section, we have analyzed the theory of CSWT, and in the following section, we will represent how to apply the CSWT to construct our shape descriptor.

4. Computation of Shape Descriptor

4.1 Pose normalization

Before the computation of the descriptor, pose normalization must be taken into consideration. Because all the models have different orientation and position in the 3D space, it is necessary to place all the models into a canonical coordinate system.

First, translate the centroid of all the models into the origin of the coordinate system.

Secondly, it is very important to obtain rotation invariance. To be sure to transform 3D objects with different orientation, D.V. Vranic [12] has done several tests, and found that it is prominent to use the PCA method to get the rotation invariant dissimilarity measures.

In our descriptor, we also use the PCA method to obtain a rotation invariant descriptor.

4.2 Spherical parameterization

In the period of spherical parameterization, a cluster of rays are cast from the centroid of the model to directions per longitude and latitude (θ_i, φ_j), $\theta_i = \frac{\pi}{2N} + i\frac{\pi}{N}$, $\varphi_j = j\frac{2\pi}{N}$, $i, j = 0,1,\ldots,N-1$, and intersect with surfaces of the model. The spherical function can be defined on the spherical grid $N \times N$ by the distances of the farthest intersections to the centroid of the model. If there is no intersection, the distance is defined as 0. And then, normalize the distances as the following.

$$d_{\theta,\varphi} = \frac{D_{\theta,\varphi}}{\sum D_{\theta,\varphi}} \tag{7}$$

Here, $D_{\theta,\varphi}$ denotes the distance between the intersection of the direction (θ,φ) and the centroid. And $\sum D_{\theta,\varphi}$ denotes the summation of all the farthest distances. Accordingly, $d_{\theta,\varphi}$ is a normalized distance. As for models with arbitary scales, they can be parameterized into the same spherical function s, as the following.

$$s(\theta,\varphi) = d_{\theta,\varphi} \tag{8}$$

where (θ,φ) denotes spherical coordinates with latitude θ and longitude φ, and $\theta \in (0,\pi]$, $\varphi \in (0,2\pi]$ respectively.

Before this section ends, there is an elucidation that N appearing in all the parts of paper has the same meaning and value as N of this section.

4.3 Discretization of CSWT of a spherical function

For a practical implementation of the CSWT, discretization is necessary. We follow the mathematic method of discretization proposed by Antoine [4] and apply the method to 3D shape retrieval algorithm.

In the algorithm, we compute the CSWT for a fixed scale

a and a fixed orientation. Specially, we adopt the value of the scale as $a = 0.05$ because the smaller scale represents the finer approximation. And the direction of the wavelet is set into 0.

As for a spherical function s, it is on the spherical grid $N \times N$ (In fact, the spherical function s is a $N \times N$ matrix and the elements are normalized distances dealt with in section 4.2), the procedure is the following.

4.3.1 Initialization

Compute the matrix $\hat{s}$ obtained by applying the FFT (Fast Fourier Transform) [18, 19] on each row of the original data s. FFT on a row is called row-wise FFT. As for a row of s, $s_i \equiv (s_{i1}, s_{i2}, ..., s_{iN})$ is transformed to $\hat{s}_i \equiv (\hat{s}_{i1}, \hat{s}_{i2}, ..., \hat{s}_{iN})$ by row-wise FFT. The sketch map is Figure 2.

$$s = \begin{bmatrix} s_{11} & \cdots & s_{1N} \\ \cdots & \cdots & \cdots \\ s_{N1} & \cdots & s_{NN} \end{bmatrix} \xrightarrow[\text{FFT}]{\text{row-wise}} \hat{s} = \begin{bmatrix} \hat{s}_{11} & \cdots & \hat{s}_{1N} \\ \cdots & \cdots & \cdots \\ \hat{s}_{N1} & \cdots & \hat{s}_{NN} \end{bmatrix}$$

Figure 2. Row-wise FFT of the spherical function

4.3.2 Production of a set of spherical wavelet basis functions

In this section, we will produce a spherical wavelet basis family, including N spherical wavelet basis functions $\psi_1, \psi_2, ..., \psi_N$. An arbitrary wavelet basis function ψ_i has a center at the spherical coordinate $(\theta_i, 0)$, called wavelet center. According to Section 3.1, the set of wavelet basis functions is constructed by rotations and dilations of a mother spherical wavelet, that is, chosen Spherical Morlet Wavelet.

As for an arbitrary wavelet basis function ψ_i, compute the value of the function on the spherical grid (θ_i, φ_j), $\theta_i = \frac{\pi}{2N} + i\frac{\pi}{N}$, $\varphi_j = j\frac{2\pi}{N}$, $i, j = 0, 1, ..., N-1$. And the result is a $N \times N$ matrix and elements of the matrix are the values of the wavelet function on the spherical grid. The value of the wavelet function on the arbitrary point $\omega = (\theta, \varphi)$ of the spherical grid can be computed as the following.

1. Calculate the rotation according to the Eq. (1). Concretely, rotate the point on the unit sphere by the angle θ_i about the x-axis. The result is that the point $\omega \equiv (\theta, \varphi)$ is transformed to the new coordinate $\omega' \equiv (\theta', \varphi')$ by the coordinate transform. In the coordinate transform, a point (x, y, z) on the unit sphere can be transformed to (x', y', z') by

$$(x', y', z') = (x, y, z) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_i & -\sin\theta_i \\ 0 & \sin\theta_i & \cos\theta_i \end{bmatrix} \tag{9}$$

Refer to the following Figure 3.

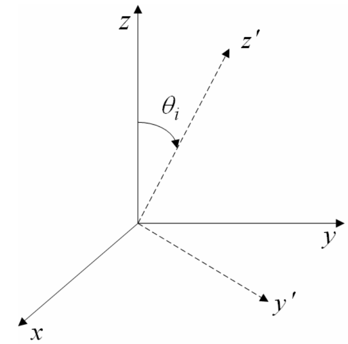


Figure 3. The coordinate transform

2. Calculate the dilation $(D_a f)(\omega') = f_a(\omega')$ according to the Eq. (2) and Eq. (6).
Here, $\omega' = (\theta', \varphi')$, compute $\lambda(a, \theta')$ according to the Eq. (3), and $f(\omega') = \psi_M(\theta', \varphi')$.

4.3.3 Row-wise FFT of a set of spherical wavelet basis functions

As for each wavelet basis function ψ_i, perform the row-wise FFT just as the section 4.3.1.

$$\psi_i = \begin{bmatrix} \psi_{11} & \cdots & \psi_{1N} \\ \cdots & \cdots & \cdots \\ \psi_{N1} & \cdots & \psi_{NN} \end{bmatrix} \xrightarrow[\text{FFT}]{\text{row-wise}} \hat{\psi}_i = \begin{bmatrix} \hat{\psi}_{11} & \cdots & \hat{\psi}_{1N} \\ \cdots & \cdots & \cdots \\ \hat{\psi}_{N1} & \cdots & \hat{\psi}_{NN} \end{bmatrix}$$

Figure 4. Row-wise FFT of a wavelet basis function

Finally, we can get the result $\hat{\psi}_1, ..., \hat{\psi}_i, ..., \hat{\psi}_N$ referring to Figure 4.

4.3.4 Medium Matrix

We can get $\hat{P}_i$ by means of the entry-by-entry multiplication of $\hat{s}$ and $\hat{\psi}_i$,

$$\hat{s} \cdot \hat{\psi}_i = \begin{bmatrix} \hat{s}_{11}\hat{\psi}_{11} & \hat{s}_{12}\hat{\psi}_{12} & \cdots & \hat{s}_{1N}\hat{\psi}_{1N} \\ \hat{s}_{21}\hat{\psi}_{21} & \hat{s}_{22}\hat{\psi}_{22} & \cdots & \hat{s}_{2N}\hat{\psi}_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{s}_{N1}\hat{\psi}_{N1} & \hat{s}_{N2}\hat{\psi}_{N2} & \cdots & \hat{s}_{NN}\hat{\psi}_{NN} \end{bmatrix}$$
$$= \begin{bmatrix} \hat{P}_{i(1,1)} & \hat{P}_{i(1,2)} & \cdots & \hat{P}_{i(1,N)} \\ \hat{P}_{i(2,1)} & \hat{P}_{i(2,2)} & \cdots & \hat{P}_{i(2,N)} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{P}_{i(N,1)} & \hat{P}_{i(N,2)} & \cdots & \hat{P}_{i(N,N)} \end{bmatrix} = \hat{\mathrm{P}}_i \tag{10}$$

followed by the row-wise Inverse Fast Fourier Transform (IFFT) [18, 19] of $\hat{P}_i$. Finally, the medium matrix P_i is created. Refer to the following sketch Figure 5.

$$\hat{P}_i = \begin{bmatrix} \hat{P}_{i(1,1)} & \hat{P}_{i(1,2)} & \cdots & \hat{P}_{i(1,N)} \\ \hat{P}_{i(2,1)} & \hat{P}_{i(2,2)} & \cdots & \hat{P}_{i(2,N)} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{P}_{i(N,1)} & \hat{P}_{i(N,2)} & \cdots & \hat{P}_{i(N,N)} \end{bmatrix} \xrightarrow[\text{IFFT}]{\text{row-wise}}$$

$$P_i = \begin{bmatrix} P_{i(1,1)} & P_{i(1,2)} & \cdots & P_{i(1,N)} \\ P_{i(2,1)} & P_{i(2,2)} & \cdots & P_{i(2,N)} \\ \cdots & \cdots & \cdots & \cdots \\ P_{i(N,1)} & P_{i(N,2)} & \cdots & P_{i(N,N)} \end{bmatrix}$$

Figure 5. Calculate the medium matrix

The results are N medium matrices P_i ($i = 1, 2, .., ...N$).

4.3.5 The result of CSWT

Using the N medium matrices P_i ($i = 1, 2, .., ...N$), we can get the value W of CSWT.

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1N} \\ w_{21} & w_{22} & \cdots & w_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ w_{N1} & w_{N2} & \cdots & w_{NN} \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \cdots \\ W_N \end{bmatrix} \qquad (11)$$

Thereinto, as for $i = 1, ..., N$, W_i is a row vector. The vector can be computed by the medium matrix P_i .

$$W_i = \begin{bmatrix} w_{i,1} & w_{i,2} & \cdots & w_{i,N} \end{bmatrix} = \varpi_i \cdot \begin{bmatrix} \sum\limits_{1 \le j \le N} P_{i(j,1)} & \cdots & \sum\limits_{1 \le j \le N} P_{i(j,k)} \cdots & \sum\limits_{1 \le j \le N} P_{i(j,N)} \end{bmatrix} \qquad (12)$$

where $P_{i(j,k)}$ is an element of the medium matrix P_i . ϖ_i is the weight, suggested in [4], and $\varpi_i = \frac{2\pi^2}{N^2} \sin(\theta_i)$.

4.4 Descriptor of shape

We adopt the result matrix W of CSWT as our descriptor of shape. The descriptor will be used for comparing shapes and discriminating shapes. The matrix has a $N \times N$ dimension.

As for two shapes a and b , the dissimilarity can be defined as the following.

$$D_S = \|W_a - W_b\|_F = (\sum\sum |w_{a,i,j} - w_{b,i,j}|^2)^{1/2} \qquad (13)$$

5. Results

In this section, we will demonstrate some results of the retrieval performance. We apply our method to a small database of general objects collected from free Princeton Shape Benchmark Database [3]. This method has been completely programmed using C++ and implemented in the environment of Visual Studio and Matlab.

In our experiment, we let $N = 64$ and sample the 64×64 directions regularly distributed on the spherical grid. And then construct the spherical function s , following with CSWT of the spherical function s . In the end, the descriptor W can be computed. The computational time complexity is $O(N^3 \log N)$.

About the shape descriptor, here give an example. As for a shape in Figure 6, we can get the descriptor as the following.

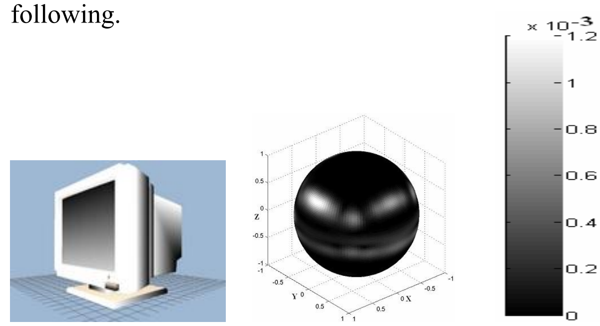
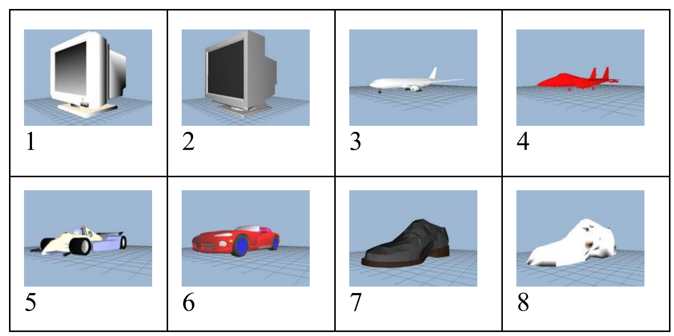


Figure 6. A shape and its descriptor

The descriptor is represented on spherical grid of the unit sphere. According to the right colorbar of Figure 6, the blackness denotes the small value and is close to 0. Oppositely, the whiteness denotes the large value.

In our experiments, we investigated the performance of our shape retrieval method based on the shape descriptor. Here, an experiment example is given. We computed the dissimilarity measure for a group of shapes. In the group of shapes, there are 8 models and 4 pairs of similar shapes, and relevant descriptors are computed, as the following Table 1.



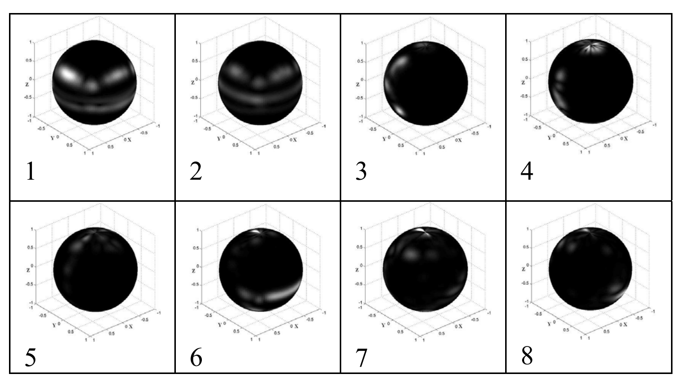


Table 1. A group of shapes and relevant descriptors

The resulting dissimilarity measures of the group of shapes are represented as a matrix in Figure 7. In the figure, the color from blackness to whiteness is proportional to the dissimilarity value from smallness to bigness, as the same as the colorbar in Figure 6. And darker elements represent a small value and similarity, and lighter elements represent a large value and dissimilarity. It is clear that the matrix is discriminating.

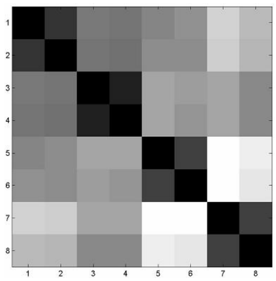


Figure 7. Dissimilarity matrix

Especially, we point out that the scale of wavelet in our descriptor can affect the performance of the retrieval, and here, we adopt the scale 0.05 to catch the finer and not coarser information in a shape. According to different sorts of models, the scale can be adjusted for a better performance of retrieval. This is one merit of this shape descriptor. Compared with other shape descriptors, for example, the shape descriptors based on spherical harmonics, this descriptor can supply a retrieval mode with more flexibility for user selection.

6. Conclusion and Future Work

In this paper, we proposed a 3D model retrieval method based on our shape descriptor made by the follow procedure. Firstly do the pose estimation followed by the spherical parameterization. Compute the CSWT of the spherical function and the result composes the descriptor. CSWT inherits the merit of wavelet analysis, and in our descriptor, can catch the important information of the shapes. And the method is confirmed to be efficient according to our experiments, and can be applied to construct a 3D retrieval system.

In addition, the computational time can be shorten by Fast CSWT algorithms suggested in [20]. We will consider this in the next step.

In the future, work concentrates on testing the 3D model retrieval method on a large of 3D models. Moreover, improve the performance of our shape descriptor and construct the more discriminating descriptor. Some efforts will be also put on researching how to build a rotation variant descriptor.

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